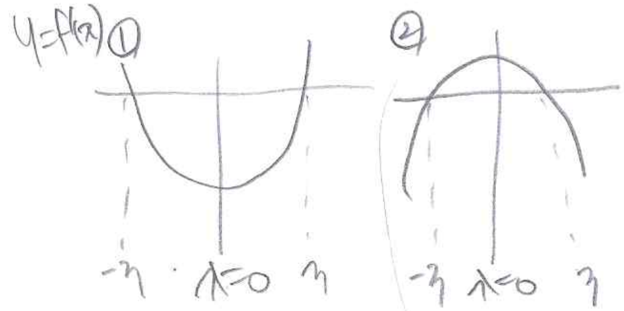
① $f(x)$ 삼차함수

가) $x=-2$. 극대

나) $f'(x)=f'(-x)$

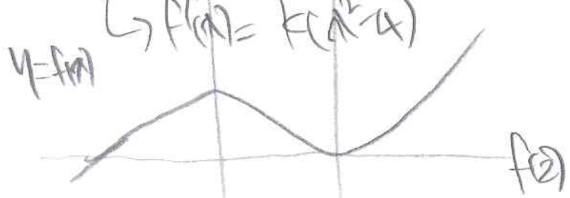
↳ $x=0$에서 축



가) $x=-2$에서 극대.

↳ $x=-2$에서 극대를 가지게 된다. (X)

↳ $f'(x)=k(x^2-4)$



ㄱ. $f'(x)$ $x=0$에서 최솟값 가진다 → (O)

ㄴ. $f(x)=f(2)$ 서로다른 두실근을 갖는다

↳ $y=f(x)$와 $y=f(2)$와의 교점 2개. (O)

ㄷ. $y=f(x)$ 위의 점 $(-1, f(-1))$에서의 접선 → $(2, f(2))$ 지난다.

$y=f'(-1)(x+1)+f(-1)$ 구하기

$y=-3k(x+1)+k\left(-\frac{1}{3}+4\right)+C$ → $y=-3kx+\frac{2}{3}k+C$

↳ $x=2$대입하면 $-\frac{16}{3}k+C$를 지난다.

$(2, f(2))$

$y'=k(x^2-4)$

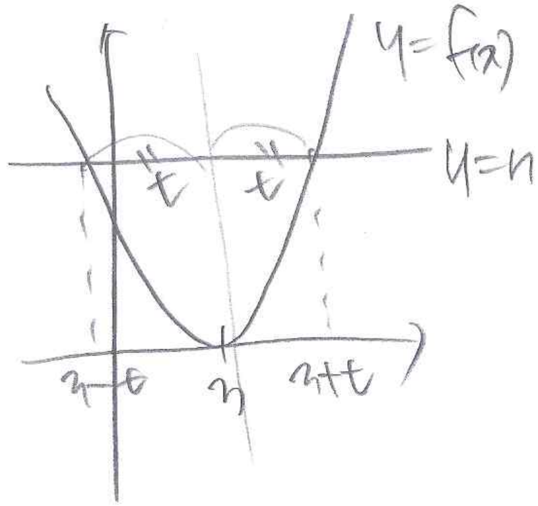
$y=k\left(\frac{1}{3}x^3-4x\right)+C$ → $\left(2, -\frac{16}{3}k+C\right)$

∴ $y=f(x)$ 위의점 $(-1, f(-1))$에서의 접선은 $(2, f(2))$를 지난다. (O)

답 ⑤

$\boxed{2}$

$f(x) = (x-a)^2$



$f(x)=n$의 두근 α, β

$h(n) = |\alpha - \beta|$ $\quad \lim_{n\to\infty} \sqrt{n}\{h(n+1)-h(n)\} = ?$

$h(n) = |\alpha-\beta| = |a+t-(a-t)|$

$= |a+t-a+t| = |2t|$

$f(a+t) = n$ 이므로 $(a+t-a)^2 = n$

$t^2 = n.$ $t = \pm\sqrt{n}$

t는 거리이므로 양수. $t = \sqrt{n}$.

$\therefore h(n) = 2\sqrt{n}$

$\lim_{n\to\infty} \sqrt{n}\{h(n+1)-h(n)\}$

$= \lim_{n\to\infty} \sqrt{n}\{2\sqrt{n+1} - 2\sqrt{n}\}$

$= \lim_{n\to\infty} \dfrac{2\sqrt{n}(\sqrt{n+1}-\sqrt{n})(\sqrt{n+1}+\sqrt{n})}{\sqrt{n+1}+\sqrt{n}}$

$= \lim_{n\to\infty} \dfrac{2\sqrt{n}}{\sqrt{n+1}+\sqrt{n}} = \dfrac{2}{1+1} = 1.$

답 ②

③ $f(x)$ 이차함수

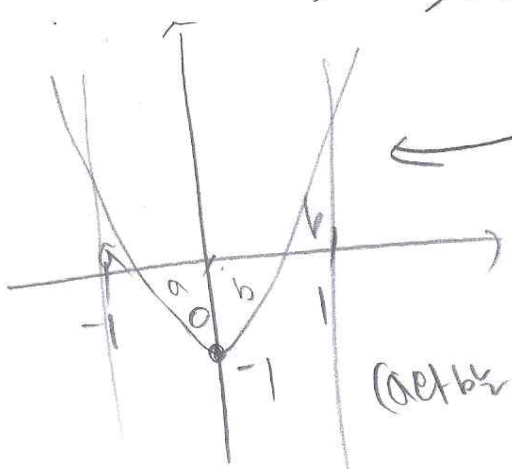
$f(0)=-1$. $\int_{-1}^{1} f(x)dx = \int_{0}^{1} f(x)dx = \int_{-1}^{0} f(x)dx = S$로 두면

$f(2)=?$

$$\int_{-1}^{1} f(x)dx = \underline{\int_{-1}^{0} f(x)dx} + \underline{\int_{0}^{1} f(x)dx}$$

$$S = S + S$$

$\therefore\ S=0$



$\int_{0}^{1} f(x)dx=0$

$\int_{-1}^{0} f(x)dx=0$

(a와 b는 x축으로 둘러싸인 넓이)

→ y축이 이차함수의 축이 된다!!

$y = kx^2-1$

$\int_{0}^{1} f(x)dx=0$ 이므로 $\int_{0}^{1} kx^2-1\,dx=0$ $\quad \left[\frac{k}{3}x^3 - x\right]_0^1=0$ $\quad \frac{k}{3}-1=0$ $\quad \therefore\ k=3$.

$f(x)=3x^2-1$

$f(2)=12-1=11$

답 ①

$\boxed{4}$ 삼차항계수 1인 삼차함수 $f(x)$

㉮ $f'(x) = f'(-x)$ (우) $\longrightarrow$ y축대칭

$y' = 3x^2 + k$

㉯ $\int_{-1}^{1} f(x)\,dx = 6$

$f'(1) = 8$이므로 $3+k=8$ $k=5$

$y' = 3x^2 + 5$

$\int_{-1}^{1} \cancel{x^3} + 5\cancel{x} + c \, dx = 6$

우:기함수적용 기함수적분 0
우함수는 $2\int_0^1 f(x)\,dx$.

$2\int_0^1 c\,dx = 6$

$c = 3$.

$\therefore f(x) = x^3 + 5x + 3$

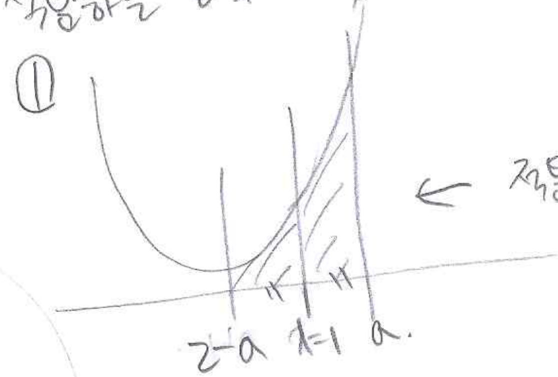
$f(1) = 1 + 5 + 3 = 9$. 답 ①

5. 최고차항계수 1 이차함수 $f(x)$

(가) 모든실수 a $\int_{2-a}^{1} f(x)\,dx = \int_{1}^{a} f(x)\,dx$

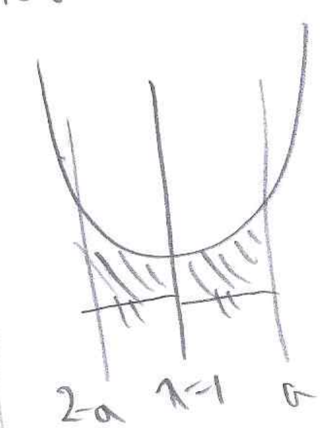
간격 $-1+a$, $a-1$

적분하는 간격이 같음 → 축은

① 

← 적분값이 같을 수가 없음

② 적분값이 같아지려면 $x=1$이 축이어야 한다

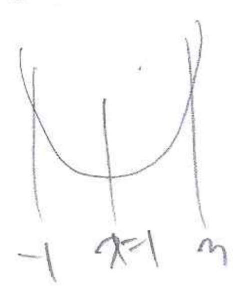


$\therefore f(x) = (x-1)^2 + k$

(나) $\int_{-1}^{3} f(x)\,dx = \frac{4}{3}$

$\int_{-1}^{3} (x-1)^2 + k \, dx = \frac{4}{3}$

$x=1$ 기준대칭이니까



$2\int_{1}^{3} (x-1)^2 + k \, dx = \frac{4}{3}$

$\int_{1}^{3} (x-1)^2 + k \, dx = \frac{2}{3}$

$k = -1$

$\therefore f(x) = (x-1)^2 - 1$

$f(4) = 8$. $\boxed{\text{답 } 8}$

$\boxed{6}$

$f(x) = (x-a)(x-b)$

$\boxed{\int_{3-t}^{3}} f(x)\,dx + \int_{3+t}^{3} f(x)\,dx = 0 \rightarrow \int_{3-t}^{3} f(x)\,dx = -\int_{3+t}^{3} f(x)\,dx$

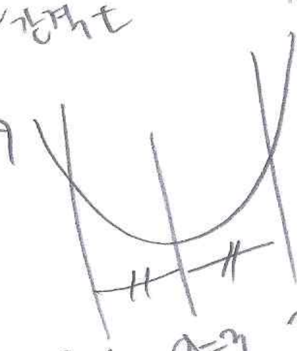
간격 t

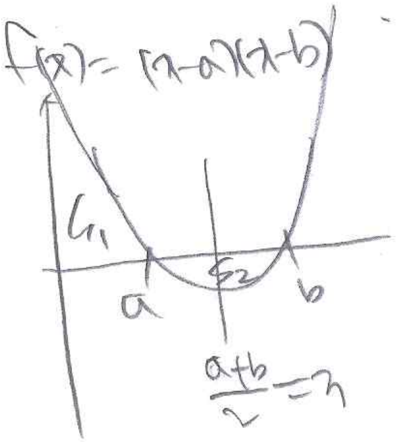
$\boxed{\int_{3-t}^{3}} f(x)\,dx = \boxed{\int_{3}^{3+t}} f(x)\,dx$

간격 t 간격 t

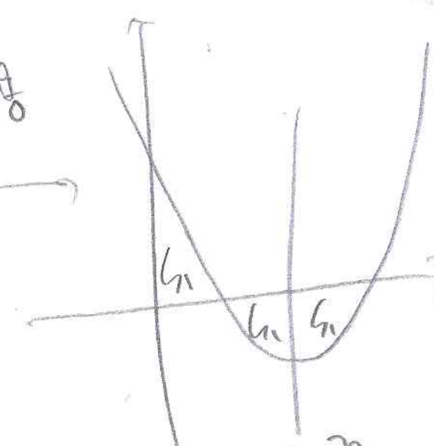
$S_2 = 2S_1$ $f(7) = ?$

$x=3$이 축이다!!





$\boxed{S_2 = 2S_1}$ 적용

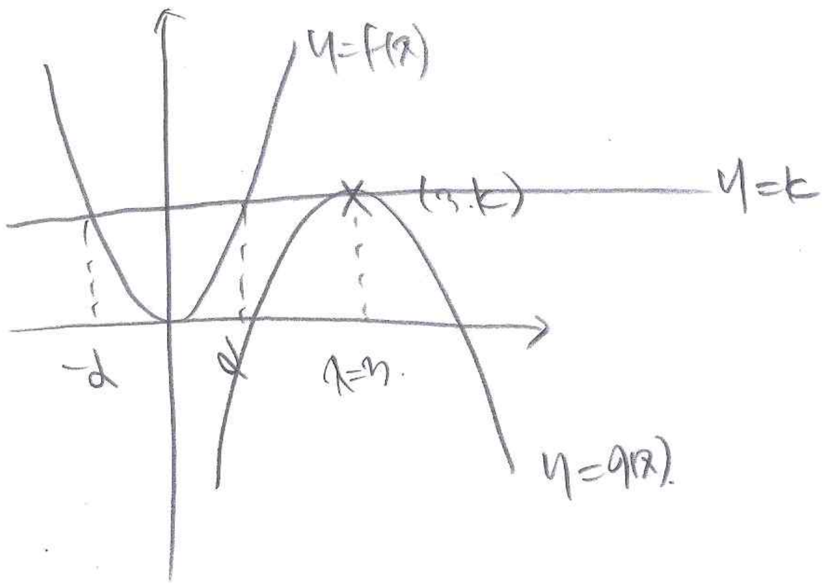


$\therefore \int_0^3 f(x)\,dx = 0$

$\int_0^3 (x-3)^2 + k\,dx = 0$

$\int_0^3 x^2 - 6x + 9 + k\,dx = 0$ $k = -3$ $\boxed{f(7) = 13}$

⑦ $f(x)=x^2$ $g(x)=-(x-3)^2+k$



$-\alpha$, α, 3 순서대로 등차수열

$2\alpha=3-\alpha$

$3\alpha=3$ $\alpha=1$

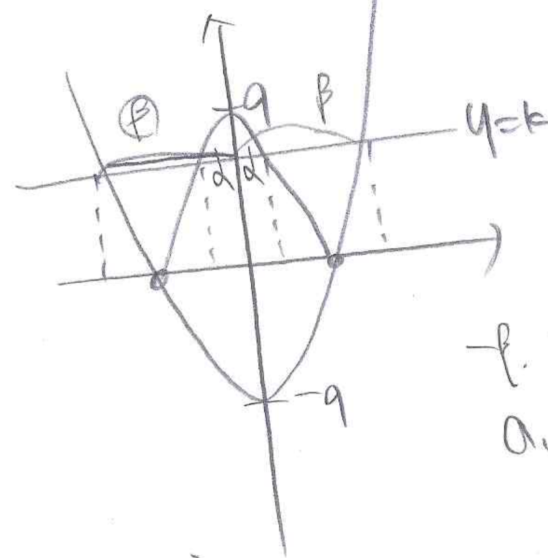
$f(1)=1$ 이므로, $k=1$.

답 ①

⑧

$y=|x^2-9|$ $y=k$. 네점에서 만난다.

a_1 a_2 a_3 a_4 등차수열



$-\beta$, $-\alpha$, α, β

a_1 a_2 a_3 a_4

간격 2α

$\therefore$ -3α $-\alpha$ α 3α

$\rightarrow$ $|(3\alpha)^2-9|=|\alpha^2-9|=k$

$9\alpha^2-9=-\alpha^2+9$

$10\alpha^2=18$

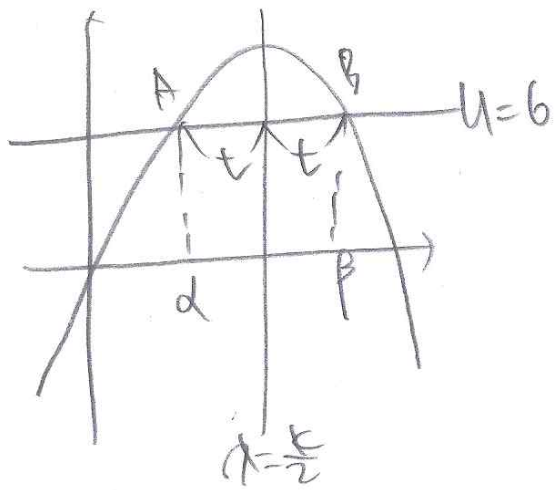
$\alpha^2=\frac{9}{5}$

$\left|\frac{9}{5}-9\right|=k$

$\frac{36}{5}=k$

답 ③

⑨ $y=-x^2+kx$



$\alpha : \frac{k}{2}-t$

$\beta : \frac{k}{2}+t$

$\frac{5}{\alpha^2}$, $\frac{1}{\alpha}+\frac{1}{\beta}$, $\frac{5}{\beta^2}$ 순서대로 등비수열

$\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^2=\frac{5}{\alpha^2}\times\frac{5}{\beta^2}$

$\left(\frac{\alpha+\beta}{\alpha\beta}\right)^2=\frac{25}{\alpha^2\beta^2}$ $(\alpha+\beta)^2=25$

$\alpha+\beta=5$ ($\because \alpha\beta$는 양수)

$\frac{k}{2}-t+\frac{k}{2}+t=5$

$k=5$

$\boxed{\text{답 } 5}$